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2003 J. Phys.: Condens. Matter 15 217

(http://iopscience.iop.org/0953-8984/15/2/321)

INSTITUTE OF PHYSICS PUBLISHING JOURNAL OF PHYSICS: CONDENSED MATTER

J. Phys.: Condens. Matter **15** (2003) 217–223 PII: S0953-8984(03)39994-1

Linear temperature dependence of the mobility in two-dimensional electron gases: many-body and spin-polarization effects

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Received 26 July 2002
Published 20 December 2002
Online at stacks.iop.org/JPhysCM/15/217

Abstract

We present results for the temperature dependence of the mobility for elastic scattering in a two-dimensional electron gas at low temperatures. Due to anomalous screening in two-dimensional systems the mobility varies linearly with temperature. We discuss many-body effects and spin-polarization effects and compare with some recent experimental and theoretical results. We show that the sign of the temperature dependence may change in spin-polarized systems.

1. Introduction

It was found in experiment [1, 2] that at low temperature the mobility in silicon metal–oxide–semiconductor structures decreases linearly with increasing temperature. The origin of this linear temperature dependence is an anomalous screening effect in two-dimensional systems as found in numerical calculations [3]. An analytical expression for the temperature-dependent mobility was derived in [4]: it was shown that the temperature dependence is due to an anomalous screening effect for wavenumbers $q \approx 2k_F$ where k_F is the Fermi wavenumber. Evidently this means that this temperature dependence of the conductivity is related to Friedel oscillations. The linear temperature dependence was calculated for weak disorder and is now known as the *ballistic regime*. The ballistic regime corresponds to the lowest-order Born approximation where the resistivity is linearly proportional to the impurity density N_i. Multiple-scattering effects are neglected.

We argued [5] that at very low temperatures a crossover in the conductivity due to interaction anomalies [6] should occur and the conductivity should decrease logarithmically with decreasing temperature. The region with logarithmic temperature dependence due to multiple-scattering effects is known as the diffusive regime. The crossover, which we predicted 15 years ago [5], was calculated recently in detail [7]. The linear temperature dependence in the ballistic regime was confirmed in this calculation. Some comments concerning this theoretical work are made at the end of this paper.

0953-8984/03/020217+07$30.00

Recently, the transport properties of two-dimensional systems in the presence of a magnetic field parallel to the two-dimensional electron gas as realized in silicon MOSFET systems have been attracted a lot of attention [8]. It was shown that the positive magnetoresistance is induced by the polarization of the spin of the electron gas [9]. This magnetoresistance was explained as a screening effect in a spin-polarized electron gas [10]. The theory has been confirmed for two-dimensional carrier systems in silicon and GaAs structures.

A lot of new experiments concerning the linear temperature dependence have been published recently including Si/Ge_xSi_{1-x} heterostructures [11, 12], Si-MOSFET systems [13–17], and $GaAs/Al_xGa_{1-x}As$ heterostructures [18]. These experiments have been made with structures having low carrier density where interaction effects are very strong. Most interesting in this context are the results in a strong parallel magnetic field (in-plane magnetic field), where the electron gas is spin polarized [11, 15–18].

In this paper we discuss the temperature dependence of the conductivity in a system with strong interaction effects and for an electron gas which is spin polarized. We shall see that in both situations many-body effects, described by a local-field correction (LFC), are essential in order to describe the transport properties. Moreover, we show that many-body effects can change the sign of the linear temperature dependence.

The paper is organized as follows. We describe the model and the theory in section 2. In section 3 we present our results for many-body and spin-polarization effects and describe the consequences for the temperature dependence of the conductivity. We also compare with experimental results. An extensive discussion of theoretical results given in [7] is in section 4. We conclude in section 5.

2. Model and theory

We consider a two-dimensional electron gas in a heterostructure with a triangular confinement potential. The two-dimensional carrier density N defines the Fermi wavenumber k_F via $N = g_s g_v k_F^2/4\pi$. g_s and g_v are the spin and the valley degeneracy, respectively. The effective Bohr radius $a^* = \varepsilon_L/m^* e^2$ is given in terms of the effective mass m^* and the dielectric constant ε_L of the host material. The Wigner–Seitz parameter r_s is determined by the electron density as $r_s^2 = 1/\pi N a^{*2}$. In this paper we present results for low density and r_s is large. The Coulomb interaction potential is given by $V(q) = 2\pi e^2 F(q)/\varepsilon_L q$. $F(q) \leqslant 1$ is the form factor due to the finite-width effects [19]. In an ideal two-dimensional electron gas width effects are neglected and $F(q) = 1$. We consider charged-impurity scattering with impurities located in the plane of the electron gas. The parameter α characterizes the behaviour of the random potential for small wavenumber $\langle|U(q \to 0|^2\rangle \propto q^{2\alpha}$ with $\alpha = -1$ for impurity scattering.

We derived the analytical result [4] for the temperature-(T-) dependent conductivity $\sigma(T)$ for *elastic* scattering for $h/\tau_0 < k_B T < \varepsilon_F$ as

$$\sigma(T) = \sigma(0)\left[1 - C(r_s, \alpha)\frac{k_B T}{\varepsilon_F}\right] \tag{1a}$$

with

$$C(r_s, \alpha) = 2C(r_s)C(\alpha). \tag{1b}$$

$\sigma(0) = Ne^2\tau_0/m$ is the conductivity at zero temperature. τ_0 is the scattering time calculated in the Born approximation, where multiple-scattering effects are neglected. The origin of this anomalous temperature dependence is the anomalous temperature dependence of the static susceptibility $\boldsymbol{X}_0(q, T)$ [19], which determines the screening function. This non-analytic behaviour of $\boldsymbol{X}_0(q, T = 0)$ at $q = 2k_F$ is related to Friedel oscillations and gives rise to the linear T dependence of the static conductivity [3, 4].

The coefficient $C(r_s)$ is given by [4]

$$C(r_s) = \frac{F(2k_F)[1 - G(2k_F)]}{F(2k_F)[1 - G(2k_F)] + 2k_F a^*/g_s g_v} \tag{2a}$$

with $k_F a^* = [4/g_s g_v]^{1/2}/r_s$. Charged-impurity scattering is characterized by

$$C(\alpha = -1) = 2\ln[2]/3 = 1.38. \tag{2b}$$

The form factor for the electron–electron interaction is expressed as

$$F(q) = \frac{1 + 9q/8b + 3q^2/8b^2}{(1 + q/b)^3}. \tag{3}$$

$1/b$ is the length parameter for the thickness of the electron gas [19]. $G(q)$ in equation (2*a*) is the LFC [20]. It describes many-body effects due to exchange and correlation beyond the random-phase approximation (RPA) and becomes extremely important in the low-density regime, where the Wigner–Seitz parameter is large. Note that equation (2*a*) describes back-scattering effects, where the wavenumber $2k_F$ enters. In a recent paper [21, 22] the LFC for two-dimensional systems (heterostructures and quantum wells) has been calculated within a sum-rule approach of the Singwi–Tosi–Land–Sjölander theory [20]. Our earlier study [4] was made within the Hubbard approximation for the LFC where only exchange is taken into account. The effect of many-body effects in quantum wells and the temperature dependence of the conductivity has been studied in [23].

For $G(2k_F) < 1$ the coefficient $C(r_s)$ is always positive, see equation (2*a*): it follows that $C(r_s, \alpha) > 0$ and this is called metallic behaviour of the conductivity. However, if $G(2k_F) > 1$ and if $2k_F a^*/g_s g_v > |F(2k_F)[1-G(2k_F)]|$ than $C(r_s)$ is negative, see equation (2*a*): it follows that $C(r_s, \alpha) < 0$ and this is called *insulating behaviour* of the conductivity.

We conclude that our theory can explain a sign-change in the temperature dependence. The sign-change is related to the LFC and therefore to the importance of many-body effects.

Equation (1) can be used to determine $C(r_s, \alpha)$ from experimental results for $\sigma(T)$: with $C(r_s, \alpha)$ from experiment and $F(2k_F)$ from theory [19] one can determine $G(2k_F)$ by using equation (2*a*). In fact, this procedure has already been used in [17] and the value of $G(2k_F)$ versus electron density was determined.

We mention that for a spin-polarized system $\sigma(0)$ in equation (1*a*) is the zero-temperature conductivity of the spin-polarized system. $\sigma(0)$ was calculated in [10]. $C(r_s, \alpha)$ for a spin-polarized system shall be discussed in section 3.

We note that in our theory a degeneracy factor $g = g_s g_v$ enters and a system with $g_s = 2$ and $g_v = 1$ is equivalent to a system with $g_s = 1$ and $g_v = 2$. Therefore, a spin-polarized electron gas in a Si-MOSFET structure with $g_v = 2$ and $g_s = 1$ is equivalent (concerning the degeneracy factor) to a non-spin-polarized electron gas in a GaAs/Al_xGa_{1-x}As heterostructure with $g_v = 1$ and $g_s = 2$.

3. Many-body and spin-polarization effects

Numerical results for $C(r_s)$ versus r_s are shown in figure 1 for a heterostructure with $g_v = 2$ and $g_s = 2$. This corresponds to a non-polarized silicon MOSFET system. The solid curve is the result including the LFC. The dashed curve is the result within the Hubbard approximation and the dotted curve is the result within the RPA, where $G(2k_F) = 0$. Note that we used a logarithmic scale. The mass anisotropy was neglected for figure 1, therefore the width effects in figure 1 are larger than in a real Si-MOSFET system. The depletion density was set to zero for the calculation shown in figure 1. The finite LFC reduces $C(r_s)$ in the given r_s-range by approximately a factor of two. We conclude that many-body effects are very important and should

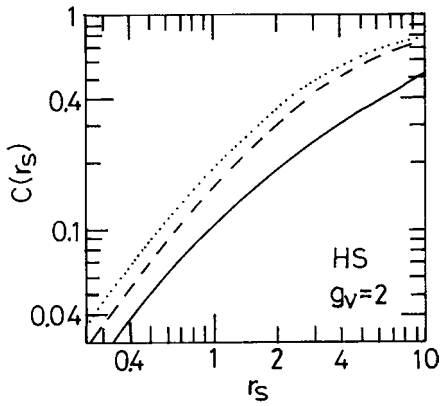

Figure 1. Coefficient $C(r_s)$ versus Wigner–Seitz parameter r_s for a heterostructure with $g_v = 2$ and $g_s = 2$ ($g = 4$). For the solid curve many-body effects are taken into account. The dashed and dotted curves represent the Hubbard approximation for the LFC and the RPA with $G(2k_F) = 0$, respectively.

be included in the calculation of the temperature dependence of transport properties. Due to exchange and correlation effects $G(2k_F)$ increases with increasing r_s, which leads to a reduced $C(r_s)$ due to correlation. Similar results have been found before for quantum wells [23].

For an ideal electron gas with $F(q) = 1$ we now describe the effects of valley degeneracy and spin degeneracy. In figure 2 we show our numerical results for $C(r_s)$ versus r_s for $g = g_v g_s = 4, 2$ and 1 as solid curves. The dashed curves correspond to the case where the LFC is neglected. This corresponds to the RPA. For $g_v g_s = 1$ we see that the coefficient $C(r_s)$ is negative at low density due to many-body effects described by $G(2k_F)$. On the other hand we see that $C(r_s)$ is positive for $g_v g_s = 2$ and 4. This means that the conductivity is metallic-like ($C(r_s) > 0$) in spin-polarized or non-polarized Si-MOSFET systems and Si/Ge_xSi_{1-x} heterostructures, as found in experiments [1, 2, 9, 11, 12, 17]. On the other hand we conclude that for low density in spin-polarized $GaAs/Al_xGa_{1-x}As$ heterostructures the conductivity is insulating-like ($C(r_s) < 0$). Exactly this behaviour was recently reported in experiment [18]. Note that $C(r_s)$ for $g = 1$ is strongly density dependent, in agreement with recent experimental results [18].

In figure 3 we show $G(2k_F)$ as a function of r_s for different values of $g = g_v g_s$. It becomes quite clear that with decreasing g the LFC becomes more and more important. At this point we must admit that our approach for the LFC is not exact and gives, for large r_s, only qualitative results. In fact, we believe that a more accurate calculation of the LFC would result in a larger $G(2k_F)$ than found within our approach, especially at large r_s [24]. We mention that the values of $G(2k_F)$ for $g = 4$ are in reasonable agreement with values obtained from recent experimental results [17].

At this point a physical interpretation is in order. The effective Coulomb interaction is given by $V_{eff}(q) = [1 - G(q)]V(q)$ and many-body effects, described by $G(q)$, decrease the effective interaction. For the spin-polarized system the effective Coulomb interaction is reduced due to the larger LFC, see figure 3. The origin of this is the Pauli principle. In fact, for a short-range interaction potential in the one-dimensional electron gas one finds $G(q) = 1$ for the spin-polarized system and interaction effects disappear completely, while $G(q) < 1$ for a non-polarized system [22]: the reason for this is that the Pauli principle does not allow us to have two electrons with the same spin at the same place in space. In one dimension

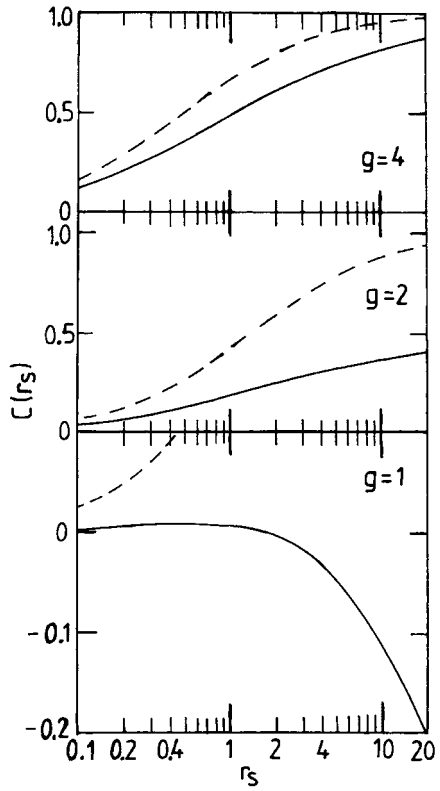

Figure 2. Coefficient $C(r_s)$ versus Wigner–Seitz parameter r_s for an ideal electron gas with no extension effects perpendicular to the interface $F(2k_F) = 1$. For the solid curves many-body effects are taken into account. Results are given for different degeneracy factors $g = g_v g_s$. The dashed curves show the RPA with $G(2k_F) = 0$.

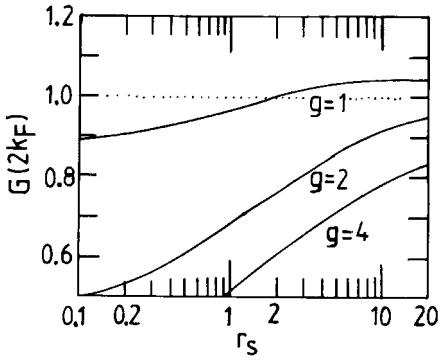

Figure 3. LFC $G(2k_F)$ versus Wigner–Seitz parameter r_s for an ideal electron gas with no extension effects perpendicular to the interface. The combined degeneracy factor $g = g_v g_s$ is indicated.

interaction effects must disappear for a short-range potential in the spin-polarized system. In two dimensions the Pauli principle leads to the enhanced LFC for $g = 1$ compared to $g = 2$ and 4, see figure 3. The fact that $G(2k_F) > 1$ for $g = 1$ means that for this wavenumber $q = 2k_F$ the effective Coulomb interaction $V_{eff}(2k_F) < 0$ is no longer repulsive but attractive.

4. Discussion

Recently, a diagrammatic approach for the temperature dependence of the conductivity has been developed in order to describe the crossover regime between the ballistic and the diffusive regime [7]. When the results are used to describe the ballistic regime a linear temperature dependence is found with a coefficient $C(r_s, \alpha)$ given by

$$C(r_s, \alpha) = -1 - \frac{3F^\sigma}{1 + F^\sigma}. \tag{4}$$

Here F^σ is a Landau Fermi liquid parameter which defines the spin susceptibility $\kappa_s = \kappa_0/(1 + F^\sigma)$ and κ_0 is the spin-susceptibility of the free electron gas.

It was argued [7] that for the spin-polarized electron gas $F^\sigma = 0$ and $C(r_s, \alpha) = -1$, independent of the carrier density. In experiment with spin-polarized holes in GaAs heterostructures [18] a negative coefficient was found; however, the coefficient was strongly density dependent, as found in our theory, see figure 2 for $g = 1$.

For $-0.25 < F^\sigma < 0$ one expects $C(r_s, \alpha) < 0$ and an *insulating* temperature dependence, in disagreement with experimental results for non-polarized electron gases [2, 12, 17]. For $F^\sigma < -0.25$ one finds $C(r_s, \alpha) > 0$ and a *metallic* temperature dependence. Within a jellium model for an interacting electron gas one expects that F^σ becomes zero for r_s going to zero (large density) [25]. Then one would expect for $F^\sigma > -0.25$ a sign change of the temperature dependence $C(r_s, \alpha) < 0$. Such a sign change as a function of the carrier density has not been observed in experiment with Si-MOSFETs [2, 12, 17].

We claim that for the non-polarized electron gas there is no predictive power in the expression given in equation (4): one uses $C(r_s, \alpha)$ as obtained from experimental results of $\sigma(T)$ to determine F^σ via equation (4). This means that the parameter $C(r_s, \alpha)$ is replaced by the parameter F^σ and one finds values for F^σ which depend on the electron density. One generally believes that the two-dimensional electron gas can be described as a Fermi liquid within a jellium model. However, it appears that the density dependence of the measured F^σ does not correspond to a jellium model [25] with F^σ approaching zero for high electron density.

However, there exist at least five problems with the diagrammatic approach of [7]. Firstly, there is a problem concerning the scattering time at zero temperature. In the diagrammatic approach contributions to the temperature independent scattering time appear which become exponentially large at low temperature, see equation (3.33) and corresponding comments in [7]. Therefore, it is fair to say that the diagrammatic approach is unable to describe the scattering time at zero temperature. In conclusion: this approach does not give a reasonable expression for τ_0.

Secondly, the linear temperature corrections to the conductivity are proportional to the scattering time τ_0. We conclude that it is difficult to give a physical sense to temperature corrections if an infinite contribution is neglected, and exactly this approach was used in [7]. In other words, I think that a theory which is unable to give a valid expression for the static conductivity at zero temperature cannot be a correct transport theory for finite temperatures.

A third problem is connected with exchange contributions [7] which are characterized by $V(q = 0)$ where $V(q)$ is the interaction potential. $V(q = 0)$ represents forward scattering. We believe that for transport properties only back-scattering $V(q = 2k_F)$ is important and $V(q = 0)$ cannot contribute. Our results [4] are obtained by taking into account only back-scattering with the correct screening function. We believe that terms of the form $V(q = 0)$ should not appear in expressions for transport properties.

The fourth problem consists in the dependence of $C(r_s, \alpha)$ on a spin parameter via F^σ, see equation (4). It is unclear why the conductivity should depend on F^σ, which is a spin parameter, while the Coulomb interaction and the random potential are independent of the

electron spin. We believe that in perturbation theory such a result is possible. However, this means that such a result is not exact.

A fifth problem is related to the form of the disorder. The results given in [7] are for a short-range random potential. The real random potential in silicon MOSFET systems for low density, where many-body effects are important, is, however, charged-impurity scattering. In [4] it was shown that $C(r_s, \alpha)$ depends on the form of the random potential. To consider disorder due to charged impurity scattering might be difficult in the theoretical frame of [7], especially in connection with the term $V(0)$.

From these arguments we conclude that we have no confidence in the approach formulated in [7]. We believe that the inclusion of LFCs in our approach goes beyond the approach of [7] and takes into account processes which are neglected in [7]. On the other hand we believe that the diagrammatic approach formulated in [7] needs more theoretical support in order to explain why the conductivity depends on a spin parameter F^{σ}. Moreover, the importance of $V(0)$ terms needs to be clarified.

5. Conclusion

The temperature dependence of the conductivity for elastic scattering in two-dimensional systems has been evaluated in the case of a low electron density and in the case of complete spin polarization. It is shown that many-body effects (exchange and correlation) are quantitatively important and can lead to a sign change in the coefficient for the linear temperature dependence in the case of a spin-polarized system. By taking many-body effects into account the present work extends the validity range of our earlier work [4] to smaller carrier density and spin-polarized systems.

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